Delayed Majority Game with Heterogeneous Learning Speeds for Financial Markets

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In this study, we introduce a delayed majority game with heterogeneous learning speeds with which stylized facts in financial markets, such as the fat tail of price returns and the power-law decaying of autocorrelation of volatility, can be reproduced. Our model is based on the Giardina-Bouchaud (GB) model [1], a delayed majority game which is an extension of Minority Game by taking the evaluation of returns of round-trip trading into account. GB model has three different regimes (a periodic, a turbulent and a stable regime) depending on two controlling parameters. The fat tail of price returns can be observed in the turbulent regime. In this model, however, market prices always follow an exponential growth as seen only in the bubble regime of the real market. Also, GB model sometimes generates an extremely large positive return which can never be observed in the real market. Last but not the least, the power-law decaying of autocorrelation of volatility is not observed in this model.

In the delayed majority game, each agent receives public information $\mu(t)$ and decides his action a (+1: buy, -1: sell, 0: do nothing) simultaneously at each discrete time step t. Agent i has S strategies and distributes score $U_{i,s}$ to s-th strategy. A strategy is an economic view which transforms public information $\mu(t)$ into action a (=1, -1 or 0). At each discrete time step t, public information $\mu(t)$ is generated and each agent decides a according to his/her highest scored strategy. Scores of strategies are updated based on the results of virtual trading. This mechanism mimics a trader who is trying to capture market dynamics by comparing hypotheses, such as fundamental, technical and momentum analysis, etc. To solve the above mentioned problems in the GB model, we construct a new model by sophisticating the GB model.

First, we simplify the GB model by omitting the interest rates and the change of the amount of agents' cash and stock. Also the definition of $\mu(t)$ is altered from bit string of the directions of price movements over previous M steps to random information. With this modification, we obtained the fat tail distribution of price changes without unrealistically large positive returns and eliminate exponential price growing, see left panel of Fig. 1.

Second, learning speeds (switching the best scored strategy) of all the agents are the same in GB model. On the other hand, there are many types of agents who have different learning speeds in real markets: some trader updates his/her trading strategies rapidly (e.g. short-term investors like day traders) and others do it more slowly (e.g. long-term investors like fundamentalists). We consider this incompleteness as the reason for missing the power-law decaying of autocorrelation of volatility in GB model. The update of strategy score is realized with a parameter β, which characterizes the learning speed of an agent: $U_{i,s}(t+1) = (1-\beta)U_{i,s}(t) + \beta a_{i,s}^{\mu(t-1)} r(t)$. ($r(t)$ is price return at time *t*. $a_{i,s}^{\mu(t-1)}$ is the action of *s*-th strategy of agent i when public information is $\mu(t-1)$.) The heterogeneity of learning speeds is introduced by dividing agents into some groups and giving a different value of β to each group. When dividing the agents into two groups, we give β = 0.55, 0.075 to each group. The learning speed is characterized by $1/\beta$, namely when β = 0.55 ($1/\beta \sim 1.8$), the characteristic time scale of learning speed corresponds to about 2 days and when β = 0.075 ($1/\beta \sim 13.3$), the time scale follows about 2 weeks. In this case, the power-law decaying of autocorrelation of volatility lasts for about only 20 steps (one step corresponds to a day). In the case of 3 groups (adding β = 0.01 which corresponds to about 3 months) and 4 groups (adding β = 0.001 corresponding to about 3 years) the power-law decaying of autocorrelation of volatility lasts for about 60 steps and for about 200 steps (right panel of Fig. 1) respectively. This improvement means that the power-law decaying of the volatility autocorrelation is somewhat related to the heterogeneity of traders' learning speeds to the previous model.

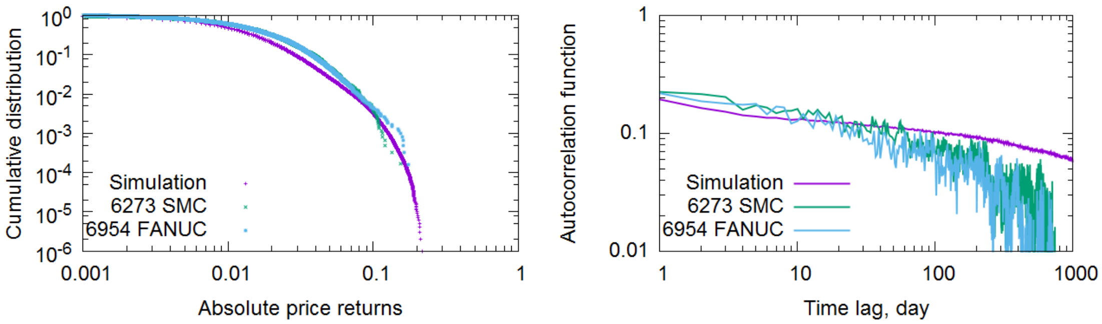

Fig. 1 The comparison of simulation results and daily stock price data of the Tokyo Stock Exchange from 4th May, 1990 to 30th December, 2015. In "6273 SMC", 6273 shows the security code and SMC is the company name. The left panel is the cumulative distribution of absolute price returns. The right panel is the autocorrelation function of volatility.

References

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