Percolation of networks with directed dependency links

Dunbiao Niu[1,2], Xin Yuan[3], Minhui Du[1], H. Eugene Stanley[3], and Yanqing Hu*[1,4]

[1] School of Data and Computer Science, Sun Yat-sen University, Guangzhou 510006, China
[2] School of Mathematics, Southwest Jiaotong University, Chengdu 610031, China
[3] Center for Polymer Studies and Department of Physics, Boston University, Boston, Massachusetts 022215 USA
[4] Big Data Research Center, University of Electronic Science and Technology of China, Chengdu 611731, China

E-mail: *yanqing.hu.sc@gmail.com

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1 Abstract

The self-consistent probabilistic approach has proven itself powerful in studying the percolation behavior of interdependent or multiplex networks without tracking the percolation process through each cascading step. In order to understand how directed dependency links impact criticality, we employ this approach to study the percolation properties of networks with both undirected connectivity links and directed dependency links. We find that when a random network with a given degree distribution undergoes a second-order phase transition, the critical point and the unstable regime surrounding the second-order phase transition regime are determined by the proportion of nodes that do not depend on any other nodes. Moreover, we also find that the triple point and the boundary between first- and second-order transitions are determined by the proportion of nodes that depend on no more than one node. This implies that it is maybe general for multiplex network systems, some important properties of phase transitions can be determined only by a few parameters. We illustrate our findings using Erdos-Renyi (ER) networks.

2 Conclusion

In summary, we present an analytical formalism for studying random networks with both connectivity links and directed dependency links under random node failures. Using a probabilistic approach, we find that the directed dependency links greatly reduce the robustness of a network. We show that the system disintegrates in a form of second-order phase transition at a critical threshold and the boundary between second-order phase transition and unstable regimes solely determined by the 6 proportion of nodes that do not depend on other nodes. Our framework also provides the solution for the boundary between the first-order and second-order phase transitions, which is characterized by the proportion of nodes that depend on no more than one node.

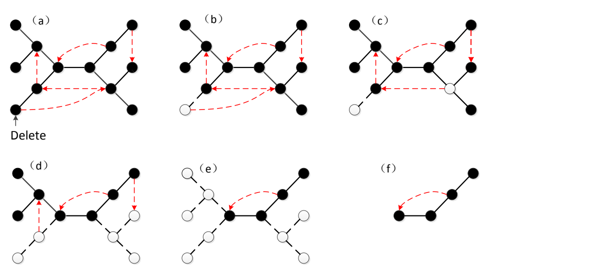

Figure1: (Color online) Demonstration of the synergy between the percolation process and the failures caused by directed dependency links that leads to a cascade of failures. The network contains two types of links: Connectivity links (solid black lines) and directed dependency links (dashed red arrows). (a)→(b) Initial failure: a random node is removed. (c)→(e) Synergy between percolation process and dependency process: Nodes cut-off from the giant component or depending on failed nodes are removed. (f) Steady state: the surviving giant component contains four nodes.

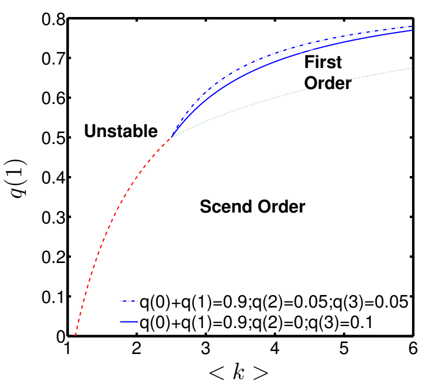

Figure2: The fraction of nodes that have one dependent node $q(1)$ as a function of the average degree $\langle k \rangle$ with $q(0)+q(1)=m_0=0.9$. The dashed lines are theoretical results obtained with intersection points at $(\frac{2}{2m_0-1},\frac{1}{2})$. The blue dashed line is the boundary between first-order phase transition and unstable system, obtained numerically. Here the red and green dashed lines only depend on m_0 whereas the blue dashed line depends on the specific details of $q(k)$ other than m_0.