Interdependent and competitive synchronization in networks of networks

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Since 2010, research on interdependent networks and networks of networks has been driven by the observation that there can exist more than one type of link in a complex system [1, 2]. For interdependent networks of networks, this is manifest in connectivity links within each network and dependency links between the networks. Nodes function only when they are connected within their network, and the node which they depend on in another network is also connected. This defines a new percolation-based concept of robustness and gives rise to distinctive phenomena including cascading failures and abrupt discontinuous transitions.

Here, we present new research on interacting network dynamics with multiple link types which exhibit dynamics that are more realistic including a wide range of new phenomena which are observed in the real-world but absent in previous models. By extending the concept of connectivity and dependency links to networks of synchronizing oscillator networks, we are able to shed light on real-world complex systems from social networks to the brain.

We begin with a standard Kuramoto model of oscillators on a network but introduce an interaction term between networks, whereby the level of local synchronization around a node in one network either increases or decreases the coupling strength around a corresponding node in another network, following Refs [3, 4]. When it increases the coupling strength, we have an interdependent interaction. When it decreases the coupling strength we have a competitive interaction. In this way, we can model a rich spectrum of interactions between oscillator networks.

References

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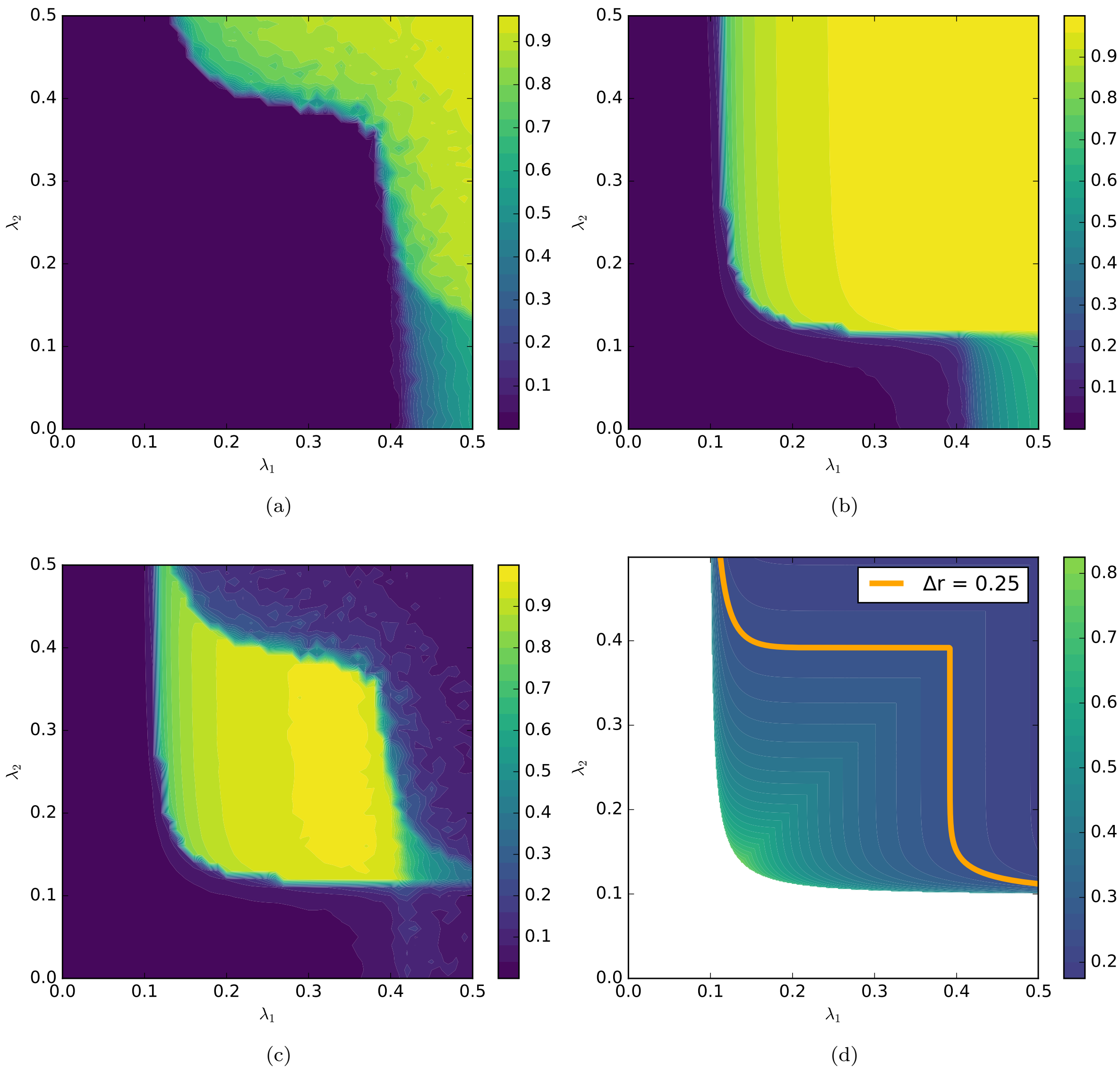

Figure1: Interdependent networks of synchronizing oscillators. **(a)** Synchronization levels starting from the fully desynchronized state. **(b)** Synchronization levels starting from the fully synchronized state. **(c)** The metastable region as determined by the differences in final synchronization levels depending on initial synchronization. **(d)** The extent of the metastable region as determined by the mean-field theory that we have developed, combined with the assumption of characteristic fluctuations of size $\Delta r = 0.25$. Numerical results obtained from a pair of coupled oscillator networks, each with $N = 65536(2^16)$ oscillators, sampled at intervals of $\Delta\lambda = 0.01$.