An inverse square law for exchange rate fluctuations ? Probability-distance based analysis relates deviation from universality with macroeconomic indicators

Abhijit Chakraborty[1], Soumya Easwaran[2], and Sitabhra Sinha[3]

The Institute of Mathematical Sciences, CIT Campus - Taramani, Chennai 600113, India

E-mail: [1]abhijitc@imsc.res.in, [2]soumyaee@imsc.res.in, [3]sitabhra@imsc.res.in

Keyword: currency exchange rates, fluctuation distribution, power law, macroeconomic indicators

The foreign exchange (FOREX) market which handles the decentralized global trade in different currencies is the largest financial market in the world. Apart from playing a key role in the international monetary and financial system, the FOREX market is a source for large volumes of data to researchers investigating financial market dynamics. We have analyzed the daily exchange rates (expressed with respect to USD) for 74 different currencies from a period of over one and a half decades (Oct 23, 1995-April 30, 2012) to observe whether the fluctuation behavior in this market exhibits universal properties ('stylized facts'). In particular, we are interested in fat-tailed behavior such as that seen in equities markets [1, 2]. To compare the fluctuation behavior of the different currencies, we have normalized the daily log returns for each currency by subtracting the mean value and dividing by their standard deviation. We observe that the normalized returns r quantifying the fluctuations in the exchange rate of currencies can appear extremely different even though they have been adjusted to have the same long-term volatility [Fig. 1 (a-b)]. The distributions of the exchange rate fluctuations for the currencies are typically heavy-tailed, with different currencies distinguished by how much leptokurtic the corresponding distributions are [Fig. 1 (c)]. We note that the power-law exponents characterizing the positive and negative long tails for the same currency need not be identical, implying that the return distribution can be asymmetric or skewed.

Analyzing the data for the set of 74 currencies we observe that the exponents γ characterizing the power-law nature of the tails of the return probability distributions have values distributed around a peak close to 3. This implies that the corresponding complementary cumulative distribution function also has a power-law form with exponent values $\alpha = \gamma - 1$ whose mean value is $\simeq 2$, suggesting an "inverse square law" governing the nature of fluctuations in the currency market analogous to the "inverse cubic law" that has been proposed as governing the price and index fluctuations in several financial markets [1, 2, 3]. As for equities, different currencies do show deviations from the average value - however, we find that there is a systematic nature for this deviation that depends on the state of the economy for the country whose currency is being considered. Thus, currencies belonging to developed economies tend to have the largest exponents, while the lowest values of exponents mostly belong to currencies from frontier economies.

To analyze this relation between exchange rate fluctuation behavior and economic condition of a country in more detail, we focus on the kurtosis of the return distributions. This quantity, defined as $\alpha_4 = E(r-\mu)^4/\sigma^4$, where $E()$ is the expectation while μ and σ are the mean and standard deviation, respectively, measures the peakedness of the distribution and is thus strongly related to its heavy tail behavior. To show the systematic nature of the deviation from a single universal, we have tried to correlate the kurtosis of the return distributions for the currencies to the macroeconomic indicators of the corresponding countries. We observe that a significant correlation exists between α_4 and the gross domestic product (GDP) per capita, g - such that, currencies of countries having higher g tend to be more stable, in the sense of having low probability of extremely large fluctuations. However, the existence of a few exceptions indicates that this is not the only factor responsible. Indeed, when we consider how the nature of the fluctuation distribution of a currency depends on the diversity of export products of the corresponding country measured by the Theil index T, we find that a significant correlation exists between α_4 and T. This indicates that, taken together, the macroeconomic factors of GDP per capita (related to the overall economic performance) and Theil index (related to the international trade of the country) explain the variation in the nature of return distributions from an universal behavior that is seen for the different currencies.

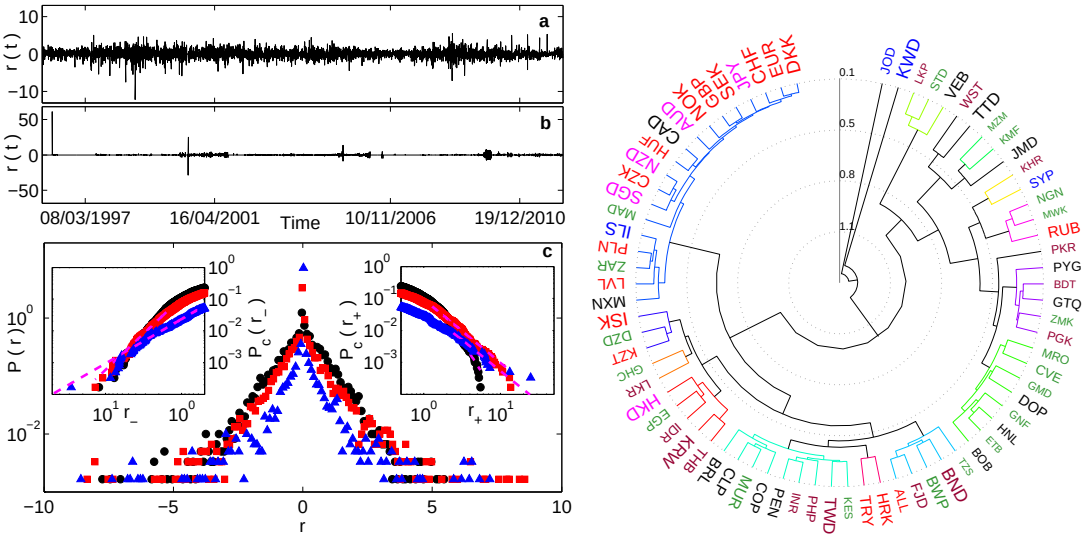

Figure1: (left) Heavy tailed behavior of currency exchange rate fluctuations. The time-series of normalized log returns $r(t)$ for currencies of developed economies, e.g., JPY (a), shows relatively lower amplitude variations compared to that of currencies of frontier economies, e.g., MZM (b), in general (note the different scales in the ordinate of the two panels). However, the distributions of r for all currencies have a heavy-tailed nature, as shown in (c) for three currencies, viz., from a developed economy, JPY (circle), an emerging economy, BRL (square), and a frontier economy, MZM (triangle). The nature of the positive (right inset) and negative tails (left inset) of the complementary cumulative distributions for these returns are also shown together with the best power-law fits (broken lines) obtained using maximum likelihood (ML) estimation. (right) Polar dendrogram representation obtained by hierarchical clustering of the different currencies in terms of the closeness of the nature of their exchange rate fluctuations. The currencies have been clustered using average linkage algorithm and the height of a branch measures the linkage function, i.e., the distance between two clusters.

We have also investigated the degree of closeness between different currencies in terms of the similarity of the nature of their fluctuation behavior. We have quantified this by measuring the distance between each pair of return probability distributions using a similarity metric based on Jensen-Shannon divergence. Hierarchical clustering of the currencies on the basis of this pairwise probability distance metric results in a grouping according to the geographical proximity of the corresponding countries and the state of the underlying economies [Fig. 1 (right)]. A time-resolved study of the data indicates that there was a major disruption during the 2008-09 financial crisis from the behavior shown in other eras, suggesting that this upheaval was qualitatively different from other recent financial market downturns and had a significant relation with the macroeconomic situation.

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