Paradoxical consequences of the inhomogenities in Bayes' rule

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Keyword: Bayes' rule, Parrondo paradox, Meaningful information extraction, Bayesian market model

Abstract

It is shown that the heterogeneities in the structure of Bayes' rule are the reason for the counterintuitive results of measurements and observations. Let's consider the simple form of Bayes' rule: $P(+|x) = \frac{P(x|+)\cdot P(+)}{P(x|+)\cdot P(+)+P(x|-)\cdot P(-)}$, and its "complement" obtained by replacing the symbol "+" with "−", where "+" and "−" are two mutually exclusive hypotheses, $P(\cdots)$ denotes the appropriate probabilities, $x \in \mathbb{N}$ or $\mathbb{R}$ the value of some random variable X associated with hypotheses "+" and "−". This rule can be expressed in one homogeneous expression: $\frac{P(+|x)}{P(-|x)} = \frac{P(+)}{P(-)} \cdot \frac{P(x|+)}{P(x|-)}$. This equation fixes the pairs $(P(+|x), P(-|x))$ and $(P(+), P(-))$ but not the pair $(P(x|+), P(x|-))$ because $P(x|+) + P(x|-) \neq 1$. These constraints reduce the number of parameters and allow us rewrite the formulas in question in the form $\frac{1+v}{1-v}$, what implies the following additive form of Bayes' rule

$$v_{|x} = \frac{v_{\pm} + v_{x|}}{1 + v_{\pm} \cdot v_{x|}}, \quad \text{where } v_{|x}, v_{x|}, v_{\pm} \in [-1, 1] \tag{1}$$

resembling the famous Einstein relativistic velocity addition formula [1]. We can introduce some additional structures in the model that results in a probability distribution on the right side of eq. (1) (eg. distinguishing between the probability measures of the measurable subsets for the set of the elementary events partition). Then, the counterintuitive behaviour of functions $P(+|x)$, $P(-|x)$ (eg. the likelihoods being linearly dependent on the left hand side of the eq. (1)) will become explicit. This would allow us to formulate a discrete Bayesian black-box like model for essential information filtering. As an important example, a "Bayesian market" model ("+" – supply, "−" – demand, $x = \ln(\text{price})$) where such inhomogenity effects are particularly evident will be presented. Such sort of market exchange can be also used for modelling transfers of information. The non-equilibrium of the market results in the existence of the non-monotonic (inconsistent with the supply/demand law) pseudo-cumulative distribution functions $P(+|x)$ and $1 - P(-|x)$ (the negative probabilities effect). In this sense, the violations of the demand and supply law should be a common phenomenon. This is another situation that leads to the anti-Parrondo effects (too much "good" produce a "bad" outcome) of which numerous examples are given in the article [2], as well as the non-computability of optimal strategy theorem for markets with dominant investor [3].

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