A quantum Brownian motion model for stock markets

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The unpredicability of stock price was expected by the notable efficient market hypothesis (EMH) and was described in the form of a Brownian motion. The idea of complete market rationality appearing in the EMH is, however, suspected by the majority nowadays. Some empirical evidence such as non-Markovianity and fat-tail distribution found in real stock markets has convinced us that the stock market cannot be described by a classical Brownian motion model (cBm(m)). It is certain that the stock market must be affected by market irrationality. Recently, a quantum damped harmonic oscillator model was introduced in order to explain why there exists persistent fluctuations of stock price. We generalize this quantum model to a scenario for quantum open systems, where we treat large numbers of stocks as a reservoir consisting of many interacting oscillators and their stock index as a free quantum Brownian particle. This new quantum Brownian motion model (qBm(m)) establishes a natural mapping between the stock market dynamics and the quantities of quantum open systems. Particularly, the persistent fluctuations in stock markets can be explained as the irrationality of stock markets and be quantified by Heisenberg's uncertainty relationship in quantum mechanics. Further analysis shows that non-Markovian autocorrelation and non-Gaussian fat-tail distribution of the return of the stock index extracted from the data of Shanghai Stock Exchange of China can be well described by quantum Brownian master equations and be explained by the quantum measurement theory. The specific expressions of non-zero kurtosis and spectral density of autocorrelation of the stock index are derived through our calculations. Within this quantum open-system framework, one is therefore able to not only interpret the dynamical behaviors of the stock market from a new statistical physcial perspective but also conduct further quantitative and empirical studies on our financial systems. Another special feature with respect to financial regulation, commonly found in Chinese stock markets, is the daily price limit. For example, daily price increase (decrease) of a stock is limited in ten percent in Shanghai Stock Exchange. We treat this special boundary condition as a periodic harmonic potential well and introduce a new quantum spatial-periodic oscillator model for daily price-limited stock markets. Considering the energy of oscillation as corresponding to trading volume, we investigate the well-known positive correlation between stock volatility and trading volume in our new model, thereby finding that the correlation becomes more complicated due to price limit. The similar volatility-volume pattern is discovered in real data of Chinese stock markets. In most cases, the limitation on the

volatility is effective, but the price limit could also abnormally increase the volatility if within a certain regime of the trading volume. This result therefore explains why unusual events with large volatility appear more often in price-limited stock markets; it also provides a new perspective to investigate the controversial topic on the advantages and disadvantages of the price limit.

References:

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