Kinetic description of the foreign exchange markets

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Social systems have attracted wide interest in terms of data analysis because of the recent development of electronic equipments. For example, a huge amount of high-precision data has been accessible in various markets, and has played important roles to analyze the market scientifically [1]. One of the accessible data in the foreign exchange markets, in particular, is the order-book data [2], which is the distribution data on limit prices. The order-books reflect the dealers' rational strategies, and are expected to be quite useful to understand their behavior scientifically. Indeed, significantly different behavior from the pure random walks has been reported on the level of the order-book distribution, and phenomenological studies are proposed on the basis of the analogy with colloidal particles in physics [3].

One of the microscopic approaches to the foreign exchange markets is to theoretically use the dealer model [4]. The dealer model is one of the multi-agent systems to predict the behavior of the market from the microscopic viewpoint. Indeed, the power-law behaviors of price differences and transaction intervals are theoretically understood from the study of the two-body stochastic dealer models [5]. However, such conventional approaches have not fully clarified the order-book distribution so far because of theoretical difficulties for the analysis of multi-agent systems. Though it is essential to address genuine many-body problems to reveal the order-book distribution, solving such problems exactly is not theoretically easy.

In this presentation, we will show a systematic theory for the order-book distribution on the basis of the kinetic theory. We first introduce a generalized stochastic dealer model and derive the corresponding hierarchy of the order-book distributions using a similar technique to the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy in the kinetic theory of molecules [6]. We then apply the truncation approximation of the two-body correlation, which we call the kinetic approximation, and derive a closed equation for the average one-body order-book distribution. We further obtain the approximate solution for the average one-body order-book distribution function, which is shown consistent with the real data in an yen-dollar foreign exchange market. We stress that our mathematical technique is quite similar to the derivation of the Boltzmann and the Vlasov equations in the kinetic theory [6], which implies that the kinetic approach developed in statistical physics is applicable even to the foreign exchange markets.

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