Detrending methods for fractional cointegration

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Detrended fluctuation analysis (DFA) [1] and detrending moving average (DMA) [2] methods are standardly used for fractional differencing parameter d (or alternatively Hurst exponent H) estimation. Both methods have been utilized for construction of scale-characteristic correlation coefficients [3, 4] that outperform standard correlation coefficient under long-range dependence [4, 5].

Recently, the DFA-based estimator of standard regression parameters has been proposed [6]. The estimator possesses some desirable properties with regards to long-range dependence, trends, seasonalities and heavy tails. The current paper contributes to the topical literature in two main ways.

First, we introduce the DMA-based estimator as a complement to the DFA-based one. And second, we study properties of both estimators beyond the general fractional cointegration framework, i.e. we examine a simple model

$$y_t = \alpha + \beta x_t + u_t,$$

where $x_t \sim I(d)$ and $u_t \sim I(d-b)$ which implies $y_t \sim I(\max[d, d-b])$. The fractional cointegration requires $b > 0$ while the standard cointegration $CI(1,1)$ assumes $x_t, y_t \sim I(1)$ and $u_t \sim I(0)$. We are interested in various combinations of d and b parameters ($0 \leq d, b \leq 1$, i.e. we cover not only the fractional cointegration framework). We provide a broad Monte Carlo simulation study focusing on different time series lengths, combination of d and b parameters, and on possible spurious relationships. Specifically, we compare the estimators based on DFA and DMA with the standard ordinary least squares (OLS) procedure under true and spurious relationships ($\beta = 0$ and $\beta \neq 0$). Based on the bias, standard error and mean squared error of the estimators, the new procedures outperform OLS for various settings (e.g. with $d = 1$ and $b < 0.5$). Detailed results will be given in the full paper.

References

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